**Learning Scenario – Simple Function Model (Vensim)**

**Basic Model:**

**Description**

This model is a graphing simulation with a function and variables that can be changed dynamically. Users may edit the value for each variable that plugs into the graphed equation and see its effect on the function’s graph. The Simple Function model is a good introduction to algebraic concepts and the idea of a function. By comparing the variables’ values and their role in the function itself, students will be able to see how each variable has an effect on the graphical trends of a function.

**Background Information**

A function is usually one of the first few interactions students have with algebra. While experience with simpler functions is recommended, it is not necessary for this model. Each variable in an equation has a specific role in its graphical representation. Users will be able to understand the versatility of functions as well as the extent that variables can have on them. The Simple Function model is simple, as the name implies, but it is not without content.

**Science/Math**

The fundamental principle behind this model is HAVE = HAD + CHANGE. Each run of the simulation includes the following steps and calculations:

1. The function is calculated based on the user-input variables
2. The function is graphed on the x-y plane

CHANGE occurs upon manipulation of the sliders, which affect the graph in real time. The function can be applied to almost any mathematical concept and real-world scenario. The relationship between the variables may be found visually on the graph.

**Teaching Strategies**

The best way to introduce this model is to review or introduce functions. It may help to draw an analogy between a “machine” and a function. A vending machine or similar input/output analogy will work. Explain to students that with each input comes a unique output. Imagine our vending machine has the following characteristics: if you put in $1, a soda comes out; if you put in $2, a snack comes out. The same input cannot produce multiple outputs. Ask the following questions to solidify their understanding of functions and the analogy:

1. If you were to put $1 into the vending machine, what would you get? Would you be able to get anything else with that $1?
2. If you were to put in $2 and you received a snack and a drink, what would that imply about the machine?
3. A machine that changes numbers into another number is called a “function.” If a function were to always add 5 to a number to produce an output, what would an input of 2 result in?
4. Would it be possible to have a function that with an input of 3 would produce an output of both 5 and 6? Explain.
5. If a function were to have multiple steps to it, such as 5\*x^2+4, how would changing any of the constants affect the output? How would the constants affect the graph of the equation?

These questions should help with a basic understanding of functions and how the constants determine the outputs. When students begin working with the model, you may want reference these questions. This should allow them to have something a connection point between what they are seeing and the ideas they already know.

**Implementation:**

**How to use the model**

Though a simple model, there are a few inputs that may be changed to produce a different graph:

1. Each of the constants (a, b, c, d) of the equation maybe changed to affect the overall behavior of the graph
2. The equation may be changed, provided it includes the constants a, b, c, and d

Both the variables and the equation may be changed by clicking on the “Equations” tool and then the variable name or “model” label. The simulation can then be run by clicking the “automatically simulate on change” button. The graph will update instantly with the new function. For more information on Vensim, reference the Vensim tutorial at: <http://shodor.org/tutorials/VensimIntroduction/Preliminaries>.

**Learning Objectives**

1. Understand how constants affect a function’s graph and the overall trends within
2. Understand the graph of a function in relation to variables with units

**Objective 1**

A function’s constants are what determine its overall shape. The default function in the model should be a\*cos(b\*x+c)+d, which is a very good equation for understanding the effects of constants. Have students click “Simulate” to get the initial graph with the default variables. Next, ask them to change the constants so that they might understand the effect that each has on the function’s shape. Ask the following questions:

1. How would you describe the overall shape of the default equation? Do you know what type of function this is?
2. Change the variable d and run the simulation again. What does the variable d change? How far detached is the constant from x in the equation? Does this separation show itself in the graph’s transformation?
3. Change each of the variables individually. What does each control in the shape of the graph? How many affect the x value, and how many affect the y?
4. Try changing the equation to a\*cos(b\*x+c)+d. Do the constants have the same effect on the sine equation as they do on the cosine?

**Objective 2**

The graph has built in units for x, time in months. The variable y can be applied to any sort of oscillating phenomenon that the teacher chooses, such as a ticking clock. Have students interpret the graph and its different points in context of the scenario. Ask the following questions to guide the students:

1. What does the variable x represent in this model? (Hint: Look at the word pointing to the variable on the worksheet).
2. The x-values are plotted on the bottom of the graph. What is the approximate position of the ticking clock at time 3? How would you explain this point on the equation in sentence format?
3. What would the range of the domain (0, 3) represent in terms of the clock’s movement?
4. Change the variable b to ¾. How does this affect the clock’s movement over time? If you were watching the clock in real-time, what would the change in b look like?
5. Would a change in variable c be possible with the clock? Explain.

**Extensions:**

1. Develop a more in-depth understanding of transformations
2. Find inverses of functions based on the outputs and inputs

**Extension 1**

The essential concept behind the Simple Function model was the effect of constants on a function, but the core of this idea relates to transformations. When one variable is changed, the whole shape of the graph can be changed. This can be broken down into simple rules and definitions. Rotations turn the whole function around a certain point, translations move functions on the graph, reflections mirror the function across a line. A function may also be shrunk and stretched. Have students research the preceding terms and how they mathematically affect equations, then apply this knowledge to the Simple Function model. Ask the following questions to help the students:

1. How does each transformational element change the function? How directly does each have to manipulate the x or y variable?
2. Change the constants to the default function in the Simple Function model. Does each affect the graph in the way that you expect? Give a name to each transformation that you do.

**Extension 2**

A function may also be reversed. That is, for every output there is a unique input. While there are methods of finding the exact equation for an inverse function, beginning students should be able to find an equation through intuition and pattern recognition. For example, the outputs 9, 4, and 1 all come from the same equation x2. The input for the function is the output for the inverse function. Therefore, since the function is x2, the output for the inverse function with an input of 9 is 3. Essentially, the inverse function is sqrt(. Students may apply this idea to both general equations and the model. Ask the following questions:

1. If a function has the outputs 9, 4, and 1, what would you expect the inputs were? Can you know for certain?
2. What is one possibility for the inputs that produced these outputs? What did you do to the outputs to find the inputs? What is the relationship between the operation you did and the original function?
3. Look at the graph of the model. If the inputs were switched with the outputs, what would the graph look like? (Hint: inputs = x, outputs = y)

**Related Models**

**Multi-Function Data Flyer**

<http://www.shodor.org/interactivate/activities/MultiFunctionDataFly/>

The Multi-Function Data Flyer allows students to plot any equation onto a graph and analyze the effects different constants have on the overall trends of the function. Essentially, it is very similar to the Simple Function model, but it allows for more complicated functions and an expansion of subject material into calculus and trigonometry. There are also multiple features that could potentially be useful to those just recently introduced to functions, such as the trace tool, which allows students to visually see all possibilities of a constant’s change at once.

**Bunny Hopping Model**

<http://www.shodor.org/talks-new/netlogo>

In the Bunny Hopping model, users are given a random sprinkling of bunnies across a hilly terrain. The bunnies naturally run downhill into the valleys. This model connects functions to topography by providing different colors and shadings based on the elevation of the terrain. Students with a basic knowledge in functions may be able to use this model to develop an understanding of relative and absolute extrema. Students may see relative minima in the valleys and an absolute minimum in the lowest point of the terrain.

**Bouncing Ball Model**

<http://www.shodor.org/talks-new/agentsheets>

Transformations, translations, and reflections do not have to simply apply to a graph of a function. The Bouncing Ball Model contains a box in which one or many balls may bounce around without diminishing velocity. Teachers may connect the graphic transformations from this model to the visual and physical representations in the Bouncing Ball Model. Since the ball bounces off the walls with perfect elasticity, obvious patterns emerge that can be connected to graphical transformations.