**Learning Scenario – Projectile Motion**

**Basic Model:**

**Description**

Most high school physics classes will cover projectile motion, but few proceed to calculate the force of air resistance and its effect on the path that the object takes in the air. This model factors in the extra variable of drag and displays the displacement, momentum, and net force over time. Initially, the force is constant, the momentum decreases linearly, and the displacement over time is parabolic. However, as velocity decreases, so too does the force of drag, which leads to a more complicated result. Students will find that they can manipulate the shapes of the graph by increasing the drag, mass, and initial momentum. This model attempts to lead students one step farther using calculus to extend the ideas of physics.

**Background Information**

In idealized physical theories like Newton's Theory of Gravity, after an object is launched from a specific height, the only force acting upon it is the force of gravity pulling it down. In actuality, gravity exists in combination with other forces, such as drag. While in classic physics problems is usually assumed that the body is relatively heavy compared with the cross-section and it is moving at a relatively low speed to avoid the issue of drag, real-life physics scenarios must factor in air resistance. Air resistance is the resulting counter-force to an object resulting from the fact that, in order to move, it must impart momentum to the air molecules in its path. According to Newton's Second Law, each action must have an equal and opposite reaction. Therefore, as the object pushes against the air molecules, the air molecules push back on the body with the same force. The Projectile Motion model accounts for this drag and allows students to see the large effect it has in changing physics models that predict the motion of an object.

**Science/Math**

The fundamental principle behind this model is HAVE = HAD + CHANGE. For every run of the simulation, the following steps occur:

1. The displacement of the simulated falling body is calculated and plotted on the graph "Falling Body" based on the variables the user inputs
2. The momentum is calculated and plotted on the graph "Momentum"
3. The net force of gravity and drag is calculated and plotted on the graph "Force"

As the above changes suggest, the CHANGE in this model is related to a specific mathematical relationship between the independent and dependent variables. Each variable that the user inputs (h, m, p, g, drag) has an effect on the overall shape of every graph. In the real world, most of these variables are roughly constant under normal circumstances, but they can be studied through observation and manipulated by changing the environment. For instance, objects held in a vacuum will not suffer drag, and so will more closely approximate a parabolic arc. Objects held on the moon or in space suffer different levels of gravitational effects, which will naturally have an impact on their acceleration.

**Teaching Strategies**

The best way to introduce this model is by asking the students questions about their overall understanding of air resistance. Begin by reviewing all the different types of forces, such as normal force, and its application to projectile motion. Then ask the following questions:

1. Hypothetically, what forces are acting on an object the moment it is launched into the air and immediately after?
2. In reality, what other forces might be acting on an object launched into the air? How do they relate to its initial velocity?
3. What is momentum? How would momentum affect an object in motion?
4. The Law of Conservation of Energy dictates that energy must remain constant, so what happens to the kinetic energy of an object if, as it moves, it imparts a force to the air molecules around it?
5. How would the shape of the graph of momentum, velocity, and net force change with and without air resistance (drag) acting on a projected object?

**Implementation:**

**How to use the model**

This relatively in-depth model has a number of parameters that can be manipulated to produce different results:

1. The initial height (x) determines how high the object starts from when it is projected into the air
2. The mass (m) is the mass of the object launched
3. The momentum (p) determines the initial velocity of the object at time 0
4. The force due to gravity (g) determines the downward force acting on the object in the air
5. The drag coefficient (drag) determines the effect of drag on the object

Each of these variables may be changed in Vensim by clicking on the "Equations" tool and clicking on each variable's respective box or circle. As soon as the variables are edited, the simulation can be run by clicking the "Simulate" button with a green triangle. Vensim will then update the graphs of displacement ("Falling Body"), momentum, and force and display them next to the model. The variables may be changed again and run multiple times.

**Learning Objectives**

1. Understand the concept of terminal velocity and its asymptotic relationship to velocity
2. Understand the effect of drag on projectile motion

**Objective 1**

Terminal velocity is related directly to the drag of an object. While in actual situations drag is determined by mass and cross-sectional shape, these are already programmed into the model, and instead the user can change the coefficient of drag. The cross-sectional shape is assumed to be fixed. Changing the coefficient of drag will change the force that drag exerts on the falling object's net force and terminal velocity. To accomplish this objective, students should change the variables to both high and low numbers in order to see the change in momentum from a linear graph to one where drag is a significant force. Ask the following questions to guide their discovery:

1. Make sure momentum (p) and drag coefficient are set to the default values of 1 kg\*m/s and 0, respectively. Run the simulation. What do the graphs of momentum and force look like? Why would they have this shape? What net forces are acting on the object?
2. Change the drag coefficient to 10. How do the graphs of momentum and net force appear different? How can you account for the change in shape—meaning what were the physical effects of changing the drag?
3. Keeping the same value for momentum, change the mass to 75 kg. How does the momentum graph change? What does it seem to be more closely imitating in terms of the shape of the graph?
4. Change the mass to 10 kg and the drag to 25. Compare it to the graph of a drag coefficient equal to 5. What happens when the coefficients grow larger? Do the graphs of momentum and force seem to approach a certain number as time progresses?
5. Compare the points at which force is close to 0 with the same points on the displacement graph. What is the shape of the "Falling Body" graph at this point and why? What does a 0 net force imply? How does this relate to Newton's Laws of Motion?

**Objective 2**

This objective focuses mainly on the effect of drag on displacement of an object. Since the net forces will counteract each other at a specific point (terminal velocity), the descent of a projectile object will be linear, with the horizontal force not being counteracted at all. The initial displacement over time graph will start as a parabolic curve, but with the effect of drag, it will become asymptotically linear. Students should manipulate the variables that control the terminal velocity point and find how terminal velocity affects an object's motion through the air. Ask the following questions:

1. Run the simulation with no drag (drag coefficient = 0) and a 1 kg object. What is the shape of the graph? Why would it take this shape? What forces are acting on the object and what is the net force?
2. Change the drag coefficient to 2. How does the displacement graph change as a result? What happens at the point that the net force becomes 0? If you were watching this object fall from the sky, describe how it would look.
3. Compare the momentum graph to the displacement graph. Why do the two seem to have a change in behavior at the same point? Explain.
4. What overall affect would you say drag has on the overall movement of an object in the air? On its momentum?

**Extensions and Related Models:**

1. Find the change in motion when the simulation is run on the moon
2. Using integrals to find the momentum of a projectile object
3. Analyze the projectile motion of Angry Birds

**Extension 1**

The Moon's force due to gravity is much less than that on earth. On Earth, gravity is -9.81 m/s2, while on the moon it is -1.66 m/s2. The Projectile Motion model can simulate launching an object on other planets with different gravitational forces. By changing the force due to gravity in the model (g) to that of the Moon's, students may be able to understand how gravity affects the motion of a projectile object. They should compare the graph of an object launched on the moon to that of one launched on Earth. Ask the following questions:

1. How does the distance traveled by the object differ on the Moon as compared to Earth? Why?
2. Would there be any drag while on the Moon? Explain.
3. In a situation where there is no drag, what effect does the mass of the falling object have on its velocity? There was a famous experiment where an Apollo astronaut dropped a hammer and a feather simultaneously while on the moon. Based on these results, which object would you expect to hit the ground first? Why?

**Extension 2**

Students may attempt to analyze the graphs of force over time in order to find the momentum of an object. Since integrals will find the area under a curve, an integral of the specific equation for the net force of the projectile object should give the momentum of the object. In this sense, momentum can be thought of as the total force exerted on the object over time. Since the model does not supply the equation of the net force used, have students graphically analyze the Net Force vs Time graph in order to find the momentum. Have them compare this number to the graph of momentum calculated in the model. Ask the following questions to ensure an understanding of their results:

1. By finding the area under the net force graph, what calculus concept are you effectively using?
2. What are the units applied to the area you found? Does this make sense compared to the graph of momentum over time? Explain.
3. How would you describe the relationship between momentum and force?

**Extension 3**

Perfect physics is not always implemented into video games. For example, in the popular mobile game Angry Birds, the gravity constant is not what it is on Earth. Use any popular physics analytics tools to track a video or image of an angry bird flying through the air. This can be done with LoggerPro or a similar tool. Have students find the force due to gravity in the Angry Birds universe and compare it to that on Earth. Follow up with the following questions:

1. What differences are there between the gravity in the Angry Birds universe and the gravity on Earth? What might be some factors that influence the Angry Birds' gravity? Does Angry Birds keep track of forces such as drag? How can you tell? Hint: what would you expect the flight path to look like if there were no drag?
2. What effects would the change in gravity have on momentum, terminal velocity, and mass, if any? Explain.

**Related Models**

**Multi-Function Data Flyer**

<http://www.shodor.org/interactivate/activities/MultiFunctionDataFly/>

The Multi-Function Data Flyer allows students to plot any equation onto a graph and analyze the effect that different constants have on the overall trends of the function. This tool can be used in multiple ways in connection with Projectile motion. Since the model can graph multiple functions at once, one possibility would be to plot the velocity, displacement, and acceleration of a projectile object over time and find the effect that initial height, drag, and mass have on its trajectory. Also, if a class were to do an experiment analyzing projectile motion, the model allows for data to be plotted on the graph and the variance to be calculated.

**Two Algorithms for Monte Carlo Integrals**

<http://www.shodor.org/talks/ncsi/excel/index.html>

The Projectile Motion model included basic activities with integrals. The Monte Carlo Integrals model allows students to extend their experience of calculating integrals to a couple different methods. Monte Carlo Integrals are calculated by finding the average area under a curve based on random points. This model will include both a description of how a Monte Carlo Integral is calculated as well as a discussion on pseudo-random numbers and their effect on computer calculations.