**Lesson Plan – Nutria**

**Basic Model:**

**Description**

 This is a simple system model of exponential population growth, with an added parameter that determines carrying capacity. The population, in this case of Nutria, a species of aquatic rodent, reproduces based on its birth rate and dies based on its death rate. The competition factor then determines to what degree death rates are determined by population. This simulates the effect of competition over limited resources and effectively limits the population.

**Background Information**

 The nutria is a large, herbivorous, semiaquatic rodent similar to the otter. They are known for breeding extremely quickly, allowing their population to rapidly explode and strip the vegetation and other resources from an area. The high fecundity of these creatures makes them an excellent choice for a population study, since even a small group can soon expand to the carrying capacity of their environment. At the carrying capacity, though, birth rates become relatively meaningless, because the death rates due to competition, predators, starvation, etc are, by definition, determined in such a way that the population does not increase.

**Science/Math**

 The fundamental principle behind this model is HAVE = HAD + CHANGE. Each time step, the new population of nutria is equal to the old population (HAD) plus the number of births and minus the number of deaths (CHANGE). Since the number of births is proportional to HAD, we would expect to see exponential growth. However, the number of deaths is also proportional to HAD and increases faster than the number of births, so we would actually expect to see the population level off at some point.

 This is a system model, so it is easy to describe with a few simple equations:

* The number of births is defined as the birth fraction times the number of nutria
* The number of deaths is defined as the number of possible pairs of nutria times the competition factor. This is an intuitive equation, because competition depends not so much on the number of nutria as on the number of interactions between nutria. The equation for this is **death = competition \* (nutria \* (nutria – 1))/2**

**Teaching Strategies**

M&M Activity:

 An effective way of introducing this model is to use a modified version of the M&M activity. For this activity, each student or group of students is given a bag of M&Ms. They start by putting two on a plate to represent the initial population. Then, in each round students will toss their M&Ms on the plate and count up the number that land "M-side" up. The population will grow by that number, so next round they will have more M&Ms to toss. This will simulate a simple model where the population grows by 50% per time step. However, students will also remove any M&Ms that are touching one another after the toss to simulate the negative effects of competition.

 The competition factor will not only reduce the rate of growth, but also reduce it in such a way that the population should roughly stabilize at a certain point. Given a finite plate size, the number of M&Ms in contact with one another will grow proportional to the total number of M&Ms squared. This means that while competition will be relatively irrelevant at first, as the population grows it will take a bigger and bigger bite until eventually it equals or outpaces reproduction.

 The competition factor in this experiment is based on plate size, so have different students or groups of students use different-size plates. After each student group has done enough trials so that the number of M&Ms is roughly constant, have them write down the final number they ended up with and compare with the rest of the class. Overall, the final population of M&Ms should be proportional to the area of the plate.

 It is unlikely that this trend will hold perfectly, particularly in a large class, so ask students to think about why this method only provides an approximation. Ask students to consider how the simulation might be different if they were dealing with extremely large areas and lots and lots of M&Ms. As the sample sizes increase, the results should begin to approach the theoretical model more and more closely.

**Implementation:**

**How to use the Model**

 This is a relatively simple population model with just a few parameters that can be changed. The important parameters are as follows:

1. The birth fraction
2. The competition factor
3. The initial number of nutria

 The birth fraction and competition factor can be changed by running the model. When the model is run, sliders will appear below these two parameters, allowing you to easily change their values. A more precise method of changing them is to right-click on the parameters before running the model and set them in the Equation box. The initial number of nutria can be set by right-clicking on the Nutria box and setting the initial number in the pop-up window that will appear. This number cannot be changed once the simulation is running.

 To run the model, click the "Automatically Simulate on Change" button. This will model the nutria population according to the given parameters for 24 months and plot the result on the graph below. The change in nutria, nutria, and death from competition values will also be graphed in their respective boxes.

**Learning Objectives:**

1. Understand why and under what circumstances the population levels off
2. Understand the relationship between recursion and exponential growth
3. Understand the difference between agent and system models

**Objective 1**

 To accomplish this objective, ask students to try out a variety of competition factors ranging from 0 to 0.01 and analyze the results. Ask the following questions:

1. Under what values for the competition factor does the population eventually level off?
2. Why do you think that even a tiny competition factor will eventually cause the population to stop growing?
3. Plot the equations y = x and y = 0.0001 \* x2 on the same graph. Does the quadratic equation ever surpass the linear equation? Why?
4. What does this imply about the stability of populations in the real world? Are all populations affected by something like a competition factor?

**Objective 2**

 To accomplish this objective, have students combine the equations for birth and death to get a single equation relating the nutria population at time *t + 1* to the population at time *t*. The resulting equation should be **nt+1 = Bnt – Cnt (nt + 1)/2**, where **B** and **C** are the birth and competition factors, respectively. This equation is difficult to solve for multiple recursions, but it can be simulated by putting it into an excel spreadsheet, where each line refers to the line before it. Have students test to verify that their equation is correct, and then ask the following questions:

1. For very small values of **n**, what is the shape of the graph? Is it similar to any other equation?
2. For larger values of **n**, what is the shape of the graph?
3. What is the mathematical explanation for these two shapes? What is the relative weight of the quadratic and linear terms for small and large values of **n**?

**Objective 3**

 To accomplish this objective, have students think about whether or not this smooth curve is an accurate representation of the real world. Ask the following questions:

1. According to this model, will the population of nutria ever decrease? Is this realistic? Why or why not?
2. If you had a population of 50 nutria in a certain environment, would you expect the population to be exactly as predicted by this model? Why or why not?
3. If you had a population of 50 million nutria in a certain, much larger, environment, would you expect the population to be as predicted by this model? Why or why not?
4. Given that the total population of nutria today is around 20,000, would you expect this model to be an accurate representation of the changes in nutria population? Why or why not?

**Extensions:**

1. Model the effect of each parameter on the overall result
2. Consider ways in which the model does not accurately match the real world

**Extension 1**

 Ask students to speculate about what would happen to the overall model if each parameter were changed. Students should write down hypotheses of the form "If parameter X goes up, then the graph will change in this way". Then, have students test their hypotheses by manipulating the parameters of the model as it runs. Ask the following questions:

1. Was your prediction correct? How do you know?
2. If your prediction were incorrect, what would the correct relationship be?
3. How does the competition parameter relate to the long-run carrying capacity?
4. How does the birth factor parameter relate to the long-run carrying capacity?
5. How does the initial population of nutria relate to the long-run carrying capacity?

**Extension 2**

 Have students consider the ways in which the Nutria model is not an accurate representation of populations in the real world. Ask the following questions:

1. What other factors might affect population of animals besides birth rates and competition? Does our model take these into account?
2. How could we add some of these extra factors into the model? What effect would you expect them to have on the results?
3. In the real world, population sometimes grows so fast that it "overshoots" its carrying capacity and must starve back down before it can reach a stable state. What are some reasons that populations might grow beyond their capacity? How could we include this factor here?

**Related Models**

**Simple Population Growth**

[www.shodor.org/talks/ncsi/excel/index.html](http://www.shodor.org/talks/ncsi/excel/index.html)

 This model is useful as a simpler example of how population would grow without a competition factor. In this model, population simply grows by a certain percentage each time step with no deaths and no possibility of decline. As a result, it will grow to infinity eventually. Students should discuss why adding a competition factor is more realistic. Since this model is also a system model, it is possible to graph a test with the same parameters in both models simultaneously and compare them to see the effect of the competition factor. The models will be similar at first, and then begin to diverge as **n** grows.

**Reaction Data**

[www.shodor.org/talks/ncsi/excel/index.html](http://www.shodor.org/talks/ncsi/excel/index.html)

This model is a great connection between population growth and other asymptotic exponential processes. Despite the fact that they are dealing with completely different concepts and have completely different derivations, the graph for concentration of a reactant over time in a completion reaction and the graph for population growth with a competition factor are remarkably similar. Students should discuss why these two processes are similar, and what we can conclude from this similarity.

Related Resources:

* Tutorial

**Epidemic**

<http://www.shodor.org/refdesk/BioPortal/model/XLepidemic?level=introductory>

 This model is another excellent connection between the population growth model and completely different fields. The epidemic model simulates the spread of a disease through a given population based on a number of factors. The simplest model, though, is the one where no one recovers or dies from the disease, and it simply spreads to everyone over time. As long as the parameters are within reasonable bounds, the disease will spread exponentially, but slowly level off over time until it reaches an asymptote, just like the population model. Have students discuss the ways in which the total population size acts in a similar way to the competition factor by limiting the maximum population of the disease.